Fractal Analysis of Image Structures

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Introduction of terms

Fractals

- Fractals are of rough or fragmented geometric shape that can be subdivided in parts, each of which is (at least approximately) a reduced copy of the whole.
- They are crinkly objects that defy conventional measures, such as length and are most often characterised by their fractal dimension
- They are mathematical sets with a high degree of geometrical complexity that can model many natural phenomena. Almost all natural objects can be observed as fractals (coastlines, trees, mountains, and clouds).
- Their fractal dimension strictly exceeds topological dimension

Fractal dimension

- The number, very often non-integer, often the only one measure of fractals
- It measures the degree of fractal boundary fragmentation or irregularity over multiple scales
- It determines how fractal differs from Euclidean objects (point, line, plane, circle etc.)

Monofractals / Multifractals

- Just a small group of fractals have one certain fractal dimension, which is scale invariant. These fractals are monofractals
- The most of natural fractals have different fractal dimensions depending on the scale. They are composed of many fractals with the different fractal dimension. They are called „multifractals“
- To characterise set of multifractals (e.g. set of the different coastlines) we do not have to establish all their fractal dimensions, it is enough to evaluate their fractal dimension at the same scale

Self-similarity/ Semi-self similarity

- Fractal is strictly self-similar if it can be expressed as a union of sets, each of which is an exactly reduced copy (is geometrically similar to) of the full set (Sierpinski triangle, Koch flake). The most fractal looking in nature do not display this precise form
- Natural objects are not union of exact reduced copies of whole. A magnified view of one part will not precisely reproduce the whole object, but it will have the same qualitative appearance. This property is called statistical self-similarity or semi-self-similarity

Box-Counting method

- One of the methods used to establish fractal dimension
- It determines the fractal dimension of black&white digitised images of fractals
- It works by covering fractal (its image) with boxes (squares) and then evaluating how many boxes are needed to cover fractal completely. Repeating this measurement with different sizes of boxes will result into logarithmical function of box size (*x*-axis) and number of boxes needed to cover fractal (*y*-axis). The slope of this function is referred as box dimension. Box dimension is taken as an appropriate approximation of fractal dimension.

Mass method/ Radius dimension method

- Further method used to establish fractal dimension
- It determines the fractal dimension of black&white digitised images of fractals too
- It is based on determination of the dependency between count of black and white pixels (picture elements) on the square (circle – radius dimension method) shaped plane, with the varying area. The slope of this dependency is the mass/radius dimension, it is a good approximation of fractal dimension. Resulting mass/radius dimension should be almost the same or the same as the box dimension.

Fractal analysis

- A collection of mathematical procedures used to determine fractal dimension (or any other fractal characteristic) or set of fractal dimensions (in the case of multifractals) with the smallest error.
- Nowadays very often used to characterise properties if natural objects
- Method under continuous scientific development

HarFA

- Software equipment to perform fractal and harmonic analysis of digitised images
- Was built up by authors of this contribution
- Is available to be freely download on http://www.fch.vutbr.cz/lectures/imagesci

Description of method

To implement Box-Counting method software called HarFA was built up. Dimension determined by this method is called Box Dimension D_{BBW}. This method has simple principle: a square mesh of various sizes $1/\varepsilon$ is laid over the image object. The count of mesh boxes N_{BBW} (ε) that contain any part of the fractal are counted (e.g. squares which are completely filled up by the fractal N_B and squares which contains just part of fractal N_{BW} are summed together). The slope of the linear portion of a function $\ln(N_B + N_{BW}) = \ln(N_{BBW}) = f(N_{BBW}(\varepsilon))$, where $\ln N_{BBW}(\varepsilon) = \ln K_{BBW} + D_{BBW}\ln(\varepsilon)$, gives D_{BBW} the fractal (box) dimension. Dimension D_{BBW} is referred as classical box dimension and can be easily find in many literature sources.

When modify this method (counting black N_B, white N_W and partially black squares N_{BW} separately) three new fractal dimensions D_B, D_W, D_{BW} can be achieved. D_B and D_W characterise fractal properties of black and white plane, while D_{BW} characterises properties of black&white border. So, we can say that HarFA can compute five independent fractal dimensions. The most important are dimensions D_{BW}, D_{BBW}, D_{WBW} (arises by summing squares N_W which are not filled up by the fractal so they remain white and squares which contains just part of fractal N_{BW}), while D_B and D_W are accidental, they are meaningful just for Euclidean objects (line, circle, square etc.). It's called **Linear Regression Analysis**.

To determine fractal dimension precisely is necessary to find linear portion of function $\ln(N_B + N_{BW}) = \ln(N_{BBW}) = f(N_{BBW}(\varepsilon))$. HarFA dispose of powerful tool to accomplish this goal. It's called **Single Slope Analysis**. Let's say that we have 100 data points. User of HarFA has to specify the length of analysed data points segment L_{DP} (e.g. 20). Then Slope Analysis sequently determines fractal dimension of data 1. - 20. next 2. -21. next 3. - 22...81. - 100. Finally we obtain the new set of fractal dimensions. If we display them on a graph (each point is colorized according appropriate correlation coefficient of linear regression) we can easily find linear portion of original function. It will exhibit by constant part on a new dependency (there are the same, or almost the same fractal dimensions) and by high correlation (marked by red or white colour).

But we have no assurance that value of L_{DP} = 20 is appropriate. So the next step is to perform Slope Analysis for all possible values of L_{DP} (from 3 to Count of Data points). This tool is called **OverAll Slope Analysis**. Its result is histogram of fractal dimension count. The most probable value of correct fractal dimension is that with the largest count. Slope Analysis provides easy form of multifractal analysis.

As said earlier, Box-Counting method works with black&white images of fractals. But medicine or biological images are mostly displayed as grey-level images or even colorized. So we need to transform these images into black&white. Procedure to accomplish this goal is called **Masking**. HarFA provides four colour spaces conversion routines (RGB, HSB/HSV, HLS and Intensity), which enables user to select desired tint intuitively. Selected tint will be transformed into black colour and all the others tints will become white. By this way black&white fractal structure arises.

But sometimes we cannot say which colour of image is important for our purposes. For these cases there is a tool called **Fractal Analysis – Range**. Fractal dimension is automatically determined for all levels of chosen channel of colour information (Red, Green, Blue, Hue, Saturation, Brightness, Intensity). Resulting fractal dimension is displayed as a function of masked level of colour information. This dependency is called **Fractal Spectrum**. It is a new and not published method of fractal analysis.

The usage of Fractal Analysis in biological or medicine sciences

As mentioned earlier, almost all natural objects can be observed as fractals. The main „beauty" of fractals consists in possibility to describe very complex natural phenomena (e.g. branching of trees or capillaries, fibrous structure of cells, clouds, cerebral cortex etc.) by small set of parameters. It's closely to the idea that the nature always prefers the simplest solution. Even so complex organism as anthill is composed of relatively simplex organisms, which execute a set of easy instructions. If you want to study fractals you can do it in two general ways: if you are an experimentalist, you try to calculate fractal dimension of things in nature and then you try to find the relationship between fractal dimension and some property of nature. If you are a theorist, you try to calculate fractal dimension of models chosen to describe experimental situations; if there is no agreement then you try another model. So the fractal dimension provides the benchmark against which theories are compared with experiments. HarFA provides the first way of solution. Authors of HarFA have documented the usage of HarFA in the root system analysis (Institut National de la Recherche Agronomique, France), the study of variation of shapes of dental crown pattern of voles (Moscow M.V.Lomonosov State University, Moscow), the analysis of cancer cells images (Gesellschaft fuer Schwerionenforschung, Germany), the plant cell identification (University of Florence, Italy) and many other usage within the different kinds of scientific interest (chemistry, physics, geography, sociology etc.).

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