The Thermodynamics of Linear Fluids and Fluid Mixtures by Pekař & Samohýl

Response to question 1

The proof which comes after (4.151) is not based on some expansion of any function. The principle will be explained on example of proof of (4.148).

Linearity of reaction rates as a function of corresponding variables is given by (4.130). This equation is result of representation of the scalar isotropic function (r_{β}) which is linear in vectors and tensors (see Appendix A.2). Note that the linearity is considered only with respect to vectors and tensors; the functional dependence on scalars (T, ρ_{γ}) is not limited. Thus, the only vectorial-tensorial independent variables on which the rates depend linearly are tr \mathbf{D}_{γ} . Their values can be chosen arbitrarily (and independently) similarly as values of the other variables in (4.140), (4.141); this selection is not limited to some small ("first order" or "linear") values. The function (4.130) is introduced - together with functions (4.131)-(4.138) - into entropic inequality and, finally, (4.139) results.

If the proved results (4.145), (4.146), and (4.154) are substituted into (4.139) then for $\mathbf{g} = \mathbf{o}$ (\mathbf{g} can be selected arbitrarily and independently) the form (A.94) follows. Next, Lemma A.5.3 is applied and (4.148) is obtained.

Thus, (4.148) is a result of the demand that entropic inequality should be valid for function (4.130) - together with functions (4.131)-(4.138) - "everywhere", i.e., also for the selection $\mathbf{g} = \mathbf{o}$ and related selection of arbitrary values of tr \mathbf{D}_{γ} . The word (third) "order" in the first paragraph on page 179 is probably misleading - it refers to the cubic term $\mathbf{u}_{\beta}^2 \operatorname{tr} \mathbf{D}_{\gamma}$ in (4.139) which, in fact, represents X^3 in (A.94) for this example. The "order" should better be replaced by "degree" here.