The Thermodynamics of Linear Fluids and Fluid Mixtures by Pekař & Samohýl

## Page 93, equation (3.92)

First, let us remind of the definition of divergence for a second-order tensor:

$$(\operatorname{div} \mathbf{A})^{i} = \sum_{k} \frac{\partial A^{ik}}{\partial x^{k}} \tag{1}$$

and a third-order tensor:

$$(\operatorname{div}\mathbf{B})^{ij} = \sum_{k} \frac{\partial B^{ijk}}{\partial x^k} \tag{2}$$

(note that the results is second-order tensor - divergence is a contraction operation).

Second, note that outer product of a vector  $(\mathbf{a})$  and a second-order tensor  $(\mathbf{A})$  is a third-order tensor with elements

$$(\mathbf{a} \wedge \mathbf{A})^{ijk} = a^i A^{jk} - a^j A^{ik}$$

and thus

$$[(\mathbf{x} - \mathbf{y}) \wedge \mathbf{T}]^{ijk} = (\mathbf{x} - \mathbf{y})^i T^{jk} - (\mathbf{x} - \mathbf{y})^j T^{ik}$$
(3)

Finally, taking into consideration (3) and (2) and employing (1):

$$\begin{aligned} \left\{ \operatorname{div}[(\mathbf{x} - \mathbf{y}) \wedge \mathbf{T}] \right\}^{ij} &= \sum_{k} \frac{\partial}{\partial x^{k}} \left[ (x - y)^{i} T^{jk} - (x - y)^{j} T^{ik} \right] \\ &= \sum_{k} \left[ T^{jk} \frac{\partial (x - y)^{i}}{\partial x^{k}} + (x - y)^{i} \frac{\partial T^{jk}}{\partial x^{k}} \right] \\ &- \sum_{k} \left[ T^{ik} \frac{\partial (x - y)^{j}}{\partial x^{k}} + (x - y)^{j} \frac{\partial T^{ik}}{\partial x^{k}} \right] \\ &= T^{ji} + (x - y)^{i} \sum_{k} \frac{\partial T^{jk}}{\partial x^{k}} - T^{ij} - (x - y)^{j} \sum_{k} \frac{\partial T^{ik}}{\partial x^{k}} \\ &\equiv T^{ji} + (x - y)^{i} (\operatorname{div} \mathbf{T})^{j} - T^{ij} - (x - y)^{j} (\operatorname{div} \mathbf{T})^{i} \\ &= T^{ji} - T^{ij} + [(\mathbf{x} - \mathbf{y}) \wedge \operatorname{div} \mathbf{T}]^{ij} \end{aligned}$$

and changing the notation to general vector-tensor symbols, (3.92) follows.