Exercise 1 to section 3.8 (and 4.7).[1] Barometric formula

Equation (3.228) enables to derive the well-known barometric formula. Find the formula and try to derive it in differential form for ideal gas before continuing reading.

The formula describes the (equilibrium) pressure change with the height above the Earth surface. In this case, there are no inertial forces ($\mathbf{i} = \mathbf{o}$) and the body forces are represented by the gravitation only: $\mathbf{b} = \mathbf{g}$, where $\mathbf{g}$ is the gravitational acceleration.

Equation (3.228) is then applied in one-dimensional form with its axis originating at the Earth surface and pointing upwards, i.e., in the opposite direction of vector $\mathbf{g}$. The component of this vector along this axis is the value of the Earth gravitational acceleration $g = \mid \mathbf{g} \mid = 9.81\,\mathrm{ms}^{-2}$. Thus:

$$\frac{\partial P}{\partial x} = -\rho g. \tag{1}$$

Equation (1) is the general (differential) form of the barometric formula. From the ideal gas state equation we derive:

$$PV = nRT = (m/M)RT = (\rho V/M)RT$$

and, consequently:

$$\rho = PM/RT. \tag{2}$$

Here m is the mass of ideal gas and M its molar mass. Substituting from (2) into (1) the final result follows:

$$\frac{\partial \ln P}{\partial x} = -\frac{Mg}{RT}.$$

[1] Based on I. Samohýl: Irreversible Thermodynamics. Prague: University of Chemical Technology, 1998 (*in Czech*).