Exercise 2 to section 4.7.[1]

Calculate the composition of air in the Earth gravitational field at the height of 8 km. Suppose that air components are oxygen and nitrogen only which behave as ideal gases and that the average air temperature is 0 °C. The oxygen content at the Earth surface is 21 molar %.

Try to answer before continuing reading.

Hint: use the Svedberg formula given on page 214, Rem. 23.

The theoretical background is the same as that for exercise 1 to section 4.7. In this exercise (Nr. 2) the theory leads to the following integral

$$\int_{0.21}^{x} \frac{dx_{\mathrm{O}_2}}{x_{\mathrm{O}_2}\Big[-x_{\mathrm{O}_2}\Big(1-\frac{M_{N_2}}{M_{O_2}}\Big)+1-\frac{M_{N_2}}{M_{O_2}}\Big]} = \int_{8000}^{0} \frac{gM_{O_2}}{RT}dz. \tag{1}$$

Integrating (1) we get

$$\Bigg[-\frac{1}{1-\frac{M_{N_2}}{M_{O_2}}}\ln\frac{-(1-\frac{M_{N_2}}{M_{O_2}})x_{O_2}+1-\frac{M_{N_2}}{M_{O_2}}}{x_{O_2}}\Bigg]_{0.21}^{x} = \Big[\frac{gM_{O_2}}{RT}z\Big]_{8000}^{0}. \tag{2}$$

From (2) we finally have an equation for our unknown quantity:

$$\ln\frac{-0.125x+0.125}{x} = -0.616\,284$$

from which $x = 0.188$; thus the oxygen concentration at 8 km is 18.8 molar %.

[1]Based on I. Samohýl: Irreversible Thermodynamics. Prague: University of Chemical Technology, 1998 (*in Czech*).