The Thermodynamics of Linear Fluids and Fluid Mixtures by Pekař \& Samohýl

## Page 120, equation (3.209)

Entropy derivatives follow from (3.208) as:

$$
\begin{align*}
& \frac{\partial \tilde{s}}{\partial T}=(1 / T)\left(\frac{\partial \tilde{u}}{\partial T}-\left(P / \rho^{2}\right) \frac{\partial \tilde{\rho}}{\partial T}\right)  \tag{1}\\
& \frac{\partial \tilde{s}}{\partial P}=(1 / T)\left(\frac{\partial \tilde{u}}{\partial P}-\left(P / \rho^{2}\right) \frac{\partial \tilde{\rho}}{\partial P}\right) . \tag{2}
\end{align*}
$$

From (1) and (2) we obtain for the second derivatives:

$$
\begin{equation*}
\frac{\partial^{2} \tilde{s}}{\partial T \partial P}=(1 / T)\left(\frac{\partial^{2} \tilde{u}}{\partial T \partial P}-\left(1 / \rho^{2}\right) \frac{\partial \tilde{\rho}}{\partial T}-\left(P / \rho^{2}\right) \frac{\partial^{2} \tilde{\rho}}{\partial T \partial P}\right) \tag{3}
\end{equation*}
$$

and

$$
\begin{align*}
\frac{\partial^{2} \tilde{s}}{\partial P \partial T} & =-\left(1 / T^{2}\right)\left(\frac{\partial \tilde{u}}{\partial P}-\left(P / \rho^{2}\right) \frac{\partial \tilde{\rho}}{\partial P}\right) \\
& +(1 / T)\left(\frac{\partial^{2} \tilde{u}}{\partial T \partial P}-\left(P / \rho^{2}\right) \frac{\partial^{2} \tilde{\rho}}{\partial P \partial T}\right) . \tag{4}
\end{align*}
$$

The derivatives in (3) and (4) should be equal, hence:

$$
\begin{aligned}
-(1 / T)\left(1 / \rho^{2}\right) \frac{\partial \rho}{\partial T} & =-\left(1 / T^{2}\right)\left(\frac{\partial \tilde{u}}{\partial P}-\left(P / \rho^{2}\right) \frac{\partial \tilde{\rho}}{\partial P}\right), \\
\left(T / \rho^{2}\right) \frac{\partial \rho}{\partial T} & =\frac{\partial \tilde{u}}{\partial P}-\left(P / \rho^{2}\right) \frac{\partial \tilde{\rho}}{\partial P} \\
\frac{\partial \tilde{u}}{\partial P} & =\left(T / \rho^{2}\right) \frac{\partial \rho}{\partial T}+\left(P / \rho^{2}\right) \frac{\partial \tilde{\rho}}{\partial P}
\end{aligned}
$$

The last equation is (3.209).

