The Thermodynamics of Linear Fluids and Fluid Mixtures by Pekař \& Samohýl
Page 121, equation (3.211)
We have from (3.63) that $\operatorname{divv}=-\dot{\rho} / \rho$. Combining this result with (3.16), we see that

$$
\begin{equation*}
\operatorname{tr} \mathbf{D}=-\frac{\dot{\rho}}{\rho} . \tag{1}
\end{equation*}
$$

From the definition (3.199) we obtain

$$
\begin{equation*}
\dot{v}=\overline{\left(\frac{-1}{\rho}\right)}=-\frac{\dot{\rho}}{\rho^{2}} \quad \Rightarrow \quad \dot{\rho}=-\rho^{2} \dot{v} . \tag{2}
\end{equation*}
$$

Inserting from (2) in (1) and using definition (3.199) we obtain:

$$
\begin{equation*}
\operatorname{tr} \mathbf{D}=\frac{\dot{v}}{v} \tag{3}
\end{equation*}
$$

Equation (3.189) with $\stackrel{\circ}{\mathbf{D}}=\mathbf{0}$ gives

$$
\begin{equation*}
\mathbf{T}_{N}=\zeta(\operatorname{tr} \mathbf{D}) \mathbf{1} \tag{4}
\end{equation*}
$$

Because the nonequilibrium pressure is defined at the considered conditions by $\mathbf{T}_{N}=-P_{N} \mathbf{1}$, it follows that $P_{N}=-\zeta(\operatorname{tr} \mathbf{D})$. Upon substitution from (3) we get $P_{N}=-\zeta(\dot{v} / v)$. Bulk viscosity $\zeta$ is a function of density and temperature (see text after (3.189)) which can be transformed to a function of the specific volume (and temperature) using (3.199). This completes derivation of (3.211).

