The Thermodynamics of Linear Fluids and Fluid Mixtures by Pekař & Samohýl

Page 121, equation (3.211)

We have from (3.63) that $\operatorname{div} \mathbf{v} = -\dot{\rho}/\rho$. Combining this result with (3.16), we see that

$$\mathrm{tr}\mathbf{D} = -\frac{\dot{\rho}}{\rho}.\tag{1}$$

From the definition (3.199) we obtain

$$\dot{v} = \left(\frac{-1}{\rho}\right) = -\frac{\dot{\rho}}{\rho^2} \qquad \Rightarrow \quad \dot{\rho} = -\rho^2 \dot{v}.$$
 (2)

Inserting from (2) in (1) and using definition (3.199) we obtain:

$$tr\mathbf{D} = \frac{\dot{v}}{v}.$$
(3)

Equation (3.189) with $\overset{\circ}{\mathbf{D}} = \mathbf{0}$ gives

$$\mathbf{T}_N = \zeta(\mathrm{tr}\mathbf{D})\mathbf{1}.\tag{4}$$

Because the nonequilibrium pressure is defined at the considered conditions by $\mathbf{T}_N = -P_N \mathbf{1}$, it follows that $P_N = -\zeta(\operatorname{tr} \mathbf{D})$. Upon substitution from (3) we get $P_N = -\zeta(\dot{v}/v)$. Bulk viscosity ζ is a function of density and temperature (see text after (3.189)) which can be transformed to a function of the specific volume (and temperature) using (3.199). This completes derivation of (3.211).