

Page 179, equation (4.148) on page 177

A slightly modified proof¹ of (4.148) is given using the definition of chemical potential (4.161). The entropy inequality is modified using (4.159), (4.161)₂, (4.150), and (4.149). Further, let us fix the nonzero vector \mathbf{u}_β and scalar $\text{tr}\mathbf{D}_\gamma$ for some constituents β and γ ($\beta, \gamma \in (1, \dots, n)$) and substitute these terms with $\lambda \mathbf{u}_\beta$ and $\lambda \text{tr}\mathbf{D}_\gamma$ in the entropy inequality. The parameter λ is a real number and all other $\mathbf{u}_\varepsilon, \text{tr}\mathbf{D}_\varepsilon$ are selected to be zeros. Finally, T, ρ_ε are selected arbitrarily but fixed, and \mathbf{g} and $\mathring{\mathbf{D}}_\varepsilon$ are set to zero ($\mathbf{g} = \mathbf{o}$ and $\mathring{\mathbf{D}}_\varepsilon = \mathbf{0}$) for all subscripts $\varepsilon = 1, \dots, n$. In this way, the entropy inequality is modified to:

$$\begin{aligned} & - \sum_{\psi=1}^{n-1} (g_\psi - g_n) r_\psi^{(0)} + \lambda \left[\rho_\gamma (g_\gamma - f_\gamma) - \sum_{\psi=1}^{n-1} (g_\psi - g_n) r_\psi^{(\gamma)} - p_\gamma \right] \text{tr}\mathbf{D}_\gamma + \\ & + \lambda^2 \left[\zeta_{\gamma\gamma} (\text{tr}\mathbf{D}_\gamma)^2 + (\nu_{\beta\beta} - \tfrac{1}{2} r_\beta^{(0)}) \mathbf{u}_\beta^2 \right] - \lambda^3 \left[\tfrac{1}{2} r_\beta^{(\gamma)} \mathbf{u}_\beta^2 \text{tr}\mathbf{D}_\gamma \right] \geq 0. \end{aligned}$$

This inequality should be valid for arbitrary values of the real parameter λ , the other quantities being constant. At sufficiently high values of λ , the last (cubic) term predominates. This term can have an arbitrary sign; therefore, the expression in it (in brackets) must be zero. This is valid for arbitrary $\text{tr}\mathbf{D}_\gamma$, arbitrary \mathbf{u}_β and also the arbitrary indexes γ, β . From this (4.148) follows for arbitrary T, ρ_1, \dots, ρ_n .

¹following the book Samohýl I. *Rational thermodynamics of chemically reacting mixtures*. Praha: Academia 1982 (in Czech).