The Thermodynamics of Linear Fluids and Fluid Mixtures by Pekař & Samohýl

Page 202, equation (4.282)

It follows from (4.281):

$$\frac{\partial \tilde{s}}{\partial T} = (1/T) \left(\frac{\partial \tilde{u}}{\partial T} - \frac{P}{\rho^2} \frac{\partial \tilde{\rho}}{\partial T} \right),\tag{1}$$

$$\frac{\partial \tilde{s}}{\partial P} = (1/T) \left(\frac{\partial \tilde{u}}{\partial P} - \frac{P}{\rho^2} \frac{\partial \tilde{\rho}}{\partial P} \right).$$
(2)

From (1) and (2) we obtain:

$$\frac{\partial^2 \tilde{s}}{\partial T \partial P} = (1/T) \frac{\partial^2 \tilde{u}}{\partial T \partial P} - (1/T) \left(\frac{P}{\rho^2} \frac{\partial^2 \tilde{\rho}}{\partial T \partial P} + \frac{1}{\rho^2} \frac{\partial \tilde{\rho}}{\partial T} \right), \tag{3}$$

$$\frac{\partial^2 \tilde{s}}{\partial P \partial T} = (-1/T^2) \left(\frac{\partial \tilde{u}}{\partial P} - \frac{P}{\rho^2} \frac{\partial \tilde{\rho}}{\partial P} \right) + (1/T) \left(\frac{\partial^2 \tilde{u}}{\partial P \partial T} - \frac{P}{\rho^2} \frac{\partial^2 \tilde{\rho}}{\partial P \partial T} \right).$$
(4)

Due to integrability condition, (3) and (4) should equal, hence

$$(-1/T\rho^2)\frac{\partial\tilde{\rho}}{\partial T} = (-1/T^2)\left(\frac{\partial\tilde{u}}{\partial P} - \frac{P}{\rho^2}\frac{\partial\tilde{\rho}}{\partial P}\right),$$
$$(T/\rho^2)\frac{\partial\tilde{\rho}}{\partial T} = \frac{\partial\tilde{u}}{\partial P} - \frac{P}{\rho^2}\frac{\partial\tilde{\rho}}{\partial P}.$$
(5)

Rearranging (5), eq. (4.282) follows immediately.