Page 202, equation (4.282)

It follows from (4.281):

$$\frac{\partial \tilde{s}}{\partial T} = (1/T)\left(\frac{\partial \tilde{u}}{\partial T} - \frac{P}{\rho^2}\frac{\partial \tilde{\rho}}{\partial T}\right), \tag{1}$$

$$\frac{\partial \tilde{s}}{\partial P} = (1/T)\left(\frac{\partial \tilde{u}}{\partial P} - \frac{P}{\rho^2}\frac{\partial \tilde{\rho}}{\partial P}\right). \tag{2}$$

From (1) and (2) we obtain:

$$\frac{\partial^2 \tilde{s}}{\partial T \partial P} = (1/T)\frac{\partial^2 \tilde{u}}{\partial T \partial P} - (1/T)\left(\frac{P}{\rho^2}\frac{\partial^2 \tilde{\rho}}{\partial T \partial P} + \frac{1}{\rho^2}\frac{\partial \tilde{\rho}}{\partial T}\right), \tag{3}$$

$$\frac{\partial^2 \tilde{s}}{\partial P \partial T} = (-1/T^2)\left(\frac{\partial \tilde{u}}{\partial P} - \frac{P}{\rho^2}\frac{\partial \tilde{\rho}}{\partial P}\right) + (1/T)\left(\frac{\partial^2 \tilde{u}}{\partial P \partial T} - \frac{P}{\rho^2}\frac{\partial^2 \tilde{\rho}}{\partial P \partial T}\right). \tag{4}$$

Due to integrability condition, (3) and (4) should equal, hence

$$(-1/T\rho^2)\frac{\partial \tilde{\rho}}{\partial T} = (-1/T^2)\left(\frac{\partial \tilde{u}}{\partial P} - \frac{P}{\rho^2}\frac{\partial \tilde{\rho}}{\partial P}\right),$$

$$(T/\rho^2)\frac{\partial \tilde{\rho}}{\partial T} = \frac{\partial \tilde{u}}{\partial P} - \frac{P}{\rho^2}\frac{\partial \tilde{\rho}}{\partial P}. \tag{5}$$

Rearranging (5), eq. (4.282) follows immediately.