The Thermodynamics of Linear Fluids and Fluid Mixtures by Pekař & Samohýl

Page 216, equation (4.344)

First, note that combining (4.30) with (4.26) we obtain:

$$\sum_{\alpha=1}^{n} S_{\sigma\alpha} J^{\alpha} = \sum_{\alpha=1}^{n} S_{\sigma\alpha} \frac{r_{\alpha}}{M_{\alpha}} = 0 \qquad \sigma = 1, ..., h.$$
(1)

Balance (4.14) gives for $\mathbf{v}_{\alpha} = \mathbf{o}$:

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_{V} \rho_{\alpha} \,\mathrm{d}v = \int_{V} r_{\alpha} \mathrm{d}v \qquad \alpha = 1, \dots, n.$$
(2)

Multiplying (2) by a constant $S_{\sigma\alpha}/M_{\alpha}$ it follows:

$$(S_{\sigma\alpha}/M_{\alpha})\frac{\mathrm{d}}{\mathrm{d}t}\int_{V}\rho_{\alpha}\,\mathrm{d}v = (S_{\sigma\alpha}/M_{\alpha})\int_{V}r_{\alpha}\mathrm{d}v \qquad \sigma = 1,...,h;\,\alpha = 1,...,n. \tag{3}$$

Summing up (3) for all constituents:

$$\sum_{\alpha=1}^{n} (S_{\sigma\alpha}/M_{\alpha}) \frac{\mathrm{d}}{\mathrm{d}t} \int_{V} \rho_{\alpha} \,\mathrm{d}v = \sum_{\alpha=1}^{n} (S_{\sigma\alpha}/M_{\alpha}) \int_{V} r_{\alpha} \mathrm{d}v,$$
$$\frac{\mathrm{d}}{\mathrm{d}t} \int_{V} \sum_{\alpha=1}^{n} (S_{\sigma\alpha}/M_{\alpha}) \rho_{\alpha} \,\mathrm{d}v = \int_{V} \sum_{\alpha=1}^{n} (S_{\sigma\alpha}/M_{\alpha}) r_{\alpha} \mathrm{d}v = 0 \qquad \sigma = 1, \dots, h \quad (4)$$

where (1) was used in the last equality.

Taking into account that $\rho_{\alpha} dv = w_{\alpha} dm$ (see the book) and multiplying (4) by E^{σ} (a constant), eq. (4.344) follows.